There is a class of TE modes which can propagate on this line.[5,6] The TE_{10} mode is the dominant mode of this structure. These TE modes should not be confused with the prior-mentioned TE modes which can only propagate in a uniformly-loaded parallel-plane line.

Tischer has shown that a conducting plane can be placed at the plane $z=0$ and not affect the field configuration of the HE_{11} mode. This conducting plane completely suppresses all of the TE modes which are symmetric about the $z=0$ plane (TE_{10}, TE_{30}, etc.). If in addition the slab width is such that

$$\frac{a}{\lambda_0} < \frac{1}{2\sqrt{\epsilon_r - 1}},$$

the TE_{20} and all higher order antisymmetric TE modes cannot propagate. The HE_{11} mode will then be the dominant mode of the resulting trough line. The attenuation due to the loss in this added conducting plane will not, however, decrease with increasing frequency.

MARVIN COHN
Radiation Lab.
The Johns Hopkins University
Baltimore, Md.

[5] M. Cohn, "Parallel plane waveguide partially filled with a dielectric," PROC. IRE, vol. 46, pp. 1952–1953; December, 1958.
[6] M. Cohn, "Propagation in a dielectric-loaded parallel plane waveguide," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 202–208; April, 1959.

Author's Comment[7]

It was interesting to learn that my proposal of 1952 for a new waveguide, the *H*-guide, found such attention, and that it became the subject of investigation at several places.

Concerning discrepancies mentioned in the above letter of Cohn, it is suggested that one should consider that approximate solutions, in general, depend on the neglections introduced and may not be called either correct or incorrect.

The equations for the attenuation of the *H*-guide dealt with in the above letter are quite complicated. One part of the attenuation constant, contributed by the wall losses, follows for comparison. The neglections, on which the approximations are based, are indicated.

The equations are valid for the waveguide of infinite height which is excited in the fundamental hybrid mode. They are approximations based on the field distribution in a lossless guide:

$$\alpha_w = \frac{R_s \left(\frac{\omega\epsilon_0\pi}{b\Gamma k_a}\right)^2}{\frac{b}{2}\left(\frac{\omega\epsilon_0}{\Gamma}\right)\left(1 + \frac{\omega^2\epsilon_0\mu_0}{k_a^2}\right)} m; \qquad (1)$$

$$m = \frac{\cos^2\psi + \frac{k_a}{k_d}\left[\psi + \sin\psi\cos\psi\right]}{\cos^2\psi + \frac{k_a}{\epsilon_r k_d}\left[\psi + \sin\psi\cos\psi\right]}, \qquad (2)$$

where

$$\psi = k_d \frac{a}{2}.$$

For small values of a, where the wall currents and the Poynting vector become approximately independent of z inside the dielectric (varying with $\cos k_d z$, where $k_d z \ll 1$), $m \to 1$, and it is the same as if the losses in the walls inside the dielectric and the power transmitted in this region were neglected.

Eq. (1) becomes, after transformation,

$$\alpha_w = \frac{2R_s}{bZ_0}\left(\frac{\lambda_0}{2b}\right)^2 \frac{1}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_{c0}}\right)^2}} \cdot \left[1 + \left(\frac{k_a\lambda_0}{2\pi}\right)^2\right]^{-1} \cdot \left[1 + \left(\frac{k_a\lambda_0}{2\pi}\right)^2\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2\right]^{-1/2}, \qquad (3)$$

which is (16) of the publication cited by Cohn.[3]

A limiting value for $\alpha_w = \alpha_0$ is obtained if $a \to 0$.

$$\alpha_0 = \frac{2R_s}{bZ_0}\left(\frac{\lambda_0}{2b}\right)^2 \frac{1}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_{c0}}\right)^2}}, \qquad (4)$$

which is the attenuation of the rectangular waveguide of infinite height excited in the TE_{10} mode. Typographical errors occurred in the above cited publication in the exponents of the equations for α_0 and (16).

Discrepancies seem still to exist between Cohn's (1) and the above (1) and (2), but no errors could be detected in our equation for α_w.

Due to lack of time and manpower in our case, I have to leave the honor of investigating and detecting this discrepancy to somebody else.

Mr. Chen Pang Wu's assistance in the calculations and checks is appreciated.

F. J. TISCHER
Dept. of Electrical Engrg.
The Ohio State University
Columbus, Ohio

[7] Received by the PGMTT, June 22, 1959.

Mr. Cohn's Reply[8]

In Dr. Tischer's above letter, I am in complete agreement with his corrected (3), since it is clearly labeled as an approximation for a thin dielectric slab. With regard to his more general equations, (1) and (2), discrepancies between our results certainly do exist. I submit that it is a necessary (though not sufficient) condition that the limiting values of α_w for $a \to 0$ and $a \to \infty$ should agree respectively with the well-known solutions for the attenuation of an infinitely-high rectangular waveguide which is air filled and dielectric filled. Eqs. (1) and (2) in Dr. Tischer's letter do not satisfy the thick slab limiting condition.

MARVIN COHN

[8] Received by the PGMTT, June 25, 1959.

Experimental Determination of Wavelength in Dielectric-Filled Periodic Structures*

Let it be required to determine the guided wavelength in a dielectric-filled periodic structure, such as a corrugated wall or serrated waveguide. The accepted travelling probe technique requires a slot in the broad wall of the guide and a groove in the dielectric material. Even if the errors introduced by these modifications could be tolerated, other effects render this technique unsuitable. One of these is a surface-wave effect which results in a measured wavelength higher than the one in the guide and lower than the free-space value. If the structure is dissipative, such as a serrated guide, more difficulties arise.

A suggested solution to this experimental problem is based on Deschamps' method of determining the elements of the scattering matrix.[1] All that is needed is the value of the argument of the transfer coefficient $S_{12}e^{i\theta}$ where $\theta + 2\pi n$ is the electrical length of the sample and n is an integer.

Suppose now that θ has been measured at two frequencies, f and $f+\delta f$, where $\delta f/f$ is of the order of 10^{-3}. Let the guided wavelengths in the two measurements be λ_g and $\lambda_g + \delta\lambda_g$, respectively, and let L be the length of the sample. Then,

$$\lambda_g(\theta + 2\pi n) = 2\pi L \qquad (1a)$$

$$(\lambda_g + \delta\lambda_g)(\theta + \delta\theta + 2\pi n) = 2\pi L \qquad (1b)$$

Implied in (1b) is the fact that θ is a continuous function of f which is true for periodic structures whose period is small compared with the guided wavelength. It appears that we have on hand two second-order equations in three unknowns: n, λ_g and $\delta\lambda_g$. But by expressing $\delta\lambda_g$ in terms of λ_g, f, and δf we can obtain one third order equation in n. It is expedient to use a method of numerical solution, because once an approximate solution has been arrived at, the exact value of n becomes the nearest integer.

We represent the structure by an equivalent dielectric constant ϵ and write

$$\lambda_g = \frac{\lambda}{\sqrt{\epsilon - \frac{\lambda^2}{\lambda_c^2}}} \qquad (2)$$

$$\lambda = \frac{c}{f} \qquad (3)$$

where λ and λ_c are the free-space and cut-off wavelength in the unloaded guide respectively.

To a first approximation we have

$$\delta\lambda_g = \frac{d\lambda_g}{d\lambda}\cdot\frac{d\lambda}{df}\cdot\delta f. \qquad (4)$$

From (2) and (3) we have

$$\frac{d\lambda_g}{d\lambda} = \left(\epsilon - \frac{\lambda^2}{\lambda_c^2}\right)^{-1/2} + \frac{\lambda^2}{\lambda_c^2}\left(\epsilon - \frac{\lambda^2}{\lambda_c^2}\right)^{-3/2}$$

and

* Manuscript received by the PGMTT, April 24, 1959; revised manuscript received, May 21, 1959.

[1] G. A. Deschamps, "Determination of reflection coefficients and insertion loss of a waveguide junction," *J. Appl. Phys.*, vol. 24, pp. 1046–1050; August, 1953.

$$\frac{d\lambda}{df} = -\frac{\lambda}{f}.$$

Substituting into (4) we obtain

$$\frac{\delta\lambda_g}{\lambda_g} = -\left(1 + \frac{\lambda_g^2}{\lambda_c^2}\right)\frac{\delta f}{f}. \quad (5)$$

Substituting again from (1a) and (5) into (1b) we obtain

$$\left\{1 - \left(1 + \frac{4\pi^2L^2}{(\theta + 2\pi n)^2\lambda_c^2}\right)\frac{\delta f}{f}\right\} \cdot \frac{2\pi L(\theta + \delta\theta + 2\pi n)}{\theta + 2\pi n} = 2\pi L.$$

Or putting

$$\theta + 2\pi n = x; \qquad \frac{4\pi^2 L^2}{\lambda_c^2} =$$

we obtain

$$\frac{\delta f}{f}x^3 - x^2\delta\theta\left(1 - \frac{\delta f}{f}\right) + x\xi\frac{\delta f}{f} + \delta\theta\xi\frac{\delta f}{f} = 0. \quad (6)$$

In view of the fact that n is an integer, no error is introduced by the approximation involved in (4). The accuracy of the method depends solely on the accuracy to which θ and L have been measured. On the other hand, f must be measured to an accuracy just sufficient for an unambiguous determination of n from (6).

Accuracy in the measurement of θ is assured by a high-quality slotted line and variable-short-circuit and it is suggested that no less than 10 points be measured for each frequency to assure the accurate plotting of the iconocentre of the transformed unit circle.

The above described method has been successfully applied to the measurement of the propagation constant in a serrated waveguide filled with polystyrene.

EFRAIM WEISSBERG
Scientific Dept.
Ministry of Defence
Hakirya, Tel Aviv, Israel

An Automatic Microwave Dielectrometer*

The dielectric constant of low-loss dielectric materials can be measured accurately, rapidly, and automatically by proper utilization of an automatic microwave impedance instrument of the type which presents its output in Smith Chart form; *i.e.*, the magnitude and phase of the reflection coefficient. The method employed is essentially a modification of the slotted line technique which was originally described by Roberts and Von Hippel,[1] the difference being that the slotted line is replaced by an automatic impedance instrument with a direct reading scale.

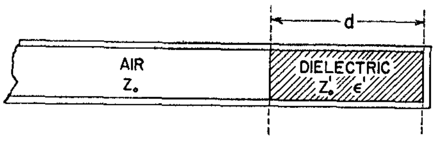

Fig. 1—Dielectric slug in short circuited section of waveguide.

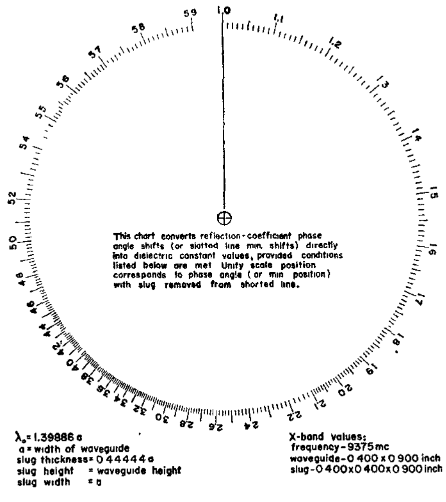

Fig. 2—Example of a direct reading dielectric constant scale.

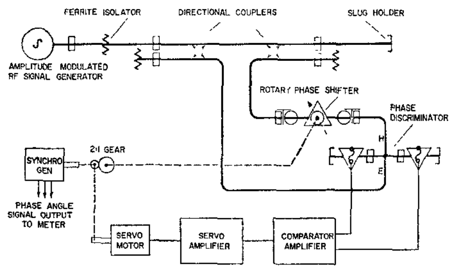

Fig. 3—Schematic diagram of an automatic dielectrometer.

A sample of the dielectric to be measured is accurately cut in the form of a slug which fills the waveguide in its transverse dimensions and is positioned against a reference short circuit, as shown in Fig. 1. From the derived relationship for this case available in the literature,[2] it will be found that the change in a slotted line minimum position, caused by insertion of the dielectric slug, can be related to the dielectric constant of the slug by

$$x = \frac{\lambda_g}{2\pi}\tan^{-1}\left[\frac{\lambda'_g}{\lambda_g}\tan\left(\frac{2\pi d}{\lambda'_g}\right)\right] - d$$

where

x = "min-shift" distance (minimum position without sample minus that with sample)
λ_g = wavelength in air-filled waveguide
λ'_g = wavelength in dielectric-filled waveguide
d = length of sample.

The particular expression for λ'_g will contain the dielectric constant, of course. Upon replacing the slotted line with an automatic impedance instrument which measures reflection coefficient, it becomes desirable to convert the "min-shift" distance, x, to the associated change in reflection coefficient phase angle, Δ_ϕ,

$$\Delta_\phi = \frac{4\pi x}{\lambda_g}.$$

One can then calculate values of Δ_ϕ for various values of dielectric constant, assuming fixed values for λ_g and d, and can construct circular scales which read directly in dielectric constant. An example of such a scale is shown in Fig. 2, calculated for 0.400 ×0.400×0.900 inch slugs in X-band waveguide at a frequency of 9375 mc. It is assumed that the relative permeability is unity and that λ'_g is given by the TE_{10} mode expression

$$\lambda'_g = \frac{\lambda_0}{\sqrt{\epsilon' - \left(\frac{\lambda_0}{2a}\right)^2}}$$

where

λ_0 = free space wavelength,
ϵ' = dielectric constant (relative)
a = width of waveguide.

Scales such as this convert reflection coefficient phase angle shifts directly into dielectric constant values, thereby permitting automatic indication. If variations in slug length or frequency are desired, a family of such scales can be constructed to cover the particular sets of conditions involved.

To use the scale, replace the Smith Chart with it on whatever output device the impedance instrument utilizes, usually an oscilloscope or a linear polar recorder. Next, align the unity scale position with the phase angle indicated for the reference short circuit by itself. Then simply insert the slugs against the short circuit and either record or read off the dielectric constant values.

The accuracy of this type of dielectrometer is dependent upon the accuracy with which the automatic impedance instrument can follow changes in reflection coefficient phase angle at a given frequency. Results comparable to those obtained from slotted-line measurements require a phase angle accuracy of about ±0.5 degrees. Instruments of this order of accuracy have been built,[3] utilizing a servo driven Fox type rotary phase shifter as the principal element in the phase measuring circuit.

If an automatic dielectrometer is desired, but an automatic impedance instrument does not happen to be available for such use, then the arrangement shown in Fig. 3 is suggested. It is based upon the reference impedance instrument.[3]

WILLIAM F. GABRIEL
Microwave Antennas and Components Branch
U. S. Naval Res. Lab.
Washington, D. C.

* Received by the PGMTT, June 12, 1959; revised, July 15, 1959.

[1] S. Riberts and A. von Hippel, "A New Method for Measuring Dielectric Constant and Loss in the Range of Centimeter Waves," Electrical Engrg. Dept., Mass. Inst. Tech., Cambridge, Mass.; March, 1941.

[2] C. G. Montgomery, "Technique of Microwave Measurements," Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 11, Ch. 10, p. 627, (71); 1947.

[3] W. F. Gabriel, "Automatic Microwave Impedance Recorder, X-Band Pototype Model," U. S. Naval Res., Lab. Rept. No. 5295, Washington, D. C.; May, 1958.